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IMPERFECT QUALITY ITEMS INVENTORY MODEL UNDER DIFFERENT DETERIORATION RATES AND SHORTAGES WITH PRICE AND TIME DEPENDENT DEMAND

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Abstract- Many times it happens that units produced or ordered are not of 100% good quality. An imperfect quality items deterministic inventory model is developed when deterioration rate is different during a cycle. Demand is a function of price and time. Shortages are allowed and completely backlogged. Numerical example is taken to support the model. Sensitivity analysis is also carried out for parameters.
Keywords – Inventory model, Varying Deterioration, Time dependent demand, Price dependent demand, Defective items, Shortages

1. INTRODUCTION
In real life effect of deterioration effect cannot be ignored for many items. With this idea Ghare and Schrader [3] developed an inventory model with constant rate of deterioration. Covert and Philip [2] extended the model by considering variable rate of deterioration. By considering shortages, the model was further extended by Shah and Jaiswal [15]. Other related work for deteriorating items are found in (Nahmias [10], Raffat [13], Goyal and Giri [4], Ouyang et al. [11], Wu et al. [18]).
Generally we assume that all received items are of good quality. But in actual practice some of the items received are not of good quality i.e. they are said to be defective items. Different inventory models have been developed for defective items in recent years. Lee and Rosenblatt [7] developed an inventory model for defective items. Cheng [1] developed a model of imperfect production quantity by establishing relationship between demand dependent unit production cost and imperfect production process. Salameh and Jaber [14] developed an inventory model in which items received are of defective quality and after 100% screening process, imperfect items are withdrawn from the inventory and sold at a discounted price. Salameh and Jaber's [14] model was extended by Wee et al. [17] by allowing shortages. Hsu and Hsu [5] considered an EOQ model for imperfect quality items with shortahe backordering and sales returns. Hauck and Voros [6] considered inventory model in which percentage of defective items as a random variable and defined the speed of the quality checking as a variable. An inventory model for one way substitutions of imperfect quality items to cope up with lost sales and shortages was developed by Mukhopadyay and Goswami [8]. Hsu and Hsu's [5] model was extended by Vishkaei et al. [16] to determine the optimal order quantity of product batches that contain defective items with percentage nonconforming following a known probability density function. Patel and Sheikh [12] developed an inventory model with different deterioration rates under price and stock dependent demand. Naik and Patel [9] developed an inventory model for imperfect quality items under price and time dependent demand.
Generally the products are such that initially there is no deterioration. Deterioration starts after certain time and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory models.
In this paper we have developed an inventory model for imperfect quality items with different deterioration rates under price and time dependent demand. Shortages are allowed. Numerical example is provided to support the model. Sensitivity analysis for major parameters is also carried out.

2. ASSUMPTIONS AND NOTATIONS
2.1 Notations:
The following notations are used for the development of the model:
D(t) : Demand rate is a function of time and price ($a+bt-\rho p$, $a>0$, $0<b<1$, $\rho>0$)
c : Purchasing cost per unit
p : Selling price per unit
d : defective items (%)

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1-d : good items (%)
λ : Screening rate
SR : Sales revenue
A : Replenishment cost per order for
z : Screening cost per unit
p_d : Price of defective items per unit
h(t) : Variable Holding cost (x + yt)
c_2 : Shortage cost per unit
t_1 : Screening time
T : Length of inventory cycle
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
Q_1 : Order quantity initially
Q_2 : Quantity of shortages
Q : Order quantity
θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta<1$
θt : Deterioration rate during , $\mu_2 \le t \le T$, $0< \theta<1$
π : Total relevant profit per unit time.

2.2 Assumptions:

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- The screening process and demand proceeds simultaneously but screening rate (λ) is greater than the demand rate i.e. $\lambda >$ (a+bt-ρp).
- The defective items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- Holding cost is time dependent.
- The screening rate (λ) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items produced or purchased.

3. THE MATHEMATICAL MODEL AND ANALYSIS

At time t=0, a lot size of Q units enters the system. Each lot having a d % defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received quantity at the rate of λ units per unit time which is greater than demand rate. After screening, a portion is used to meet the backlogging items towards previous shortages and initial inventory for period is Q_1. During the screening process the demand occurs parallel to the screening process and is fulfilled from goods which are found to be of perfect quality by the screening process. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price. After the screening process at time t_1 the inventory level will be I(t) and at time t_0, inventory level will become zero due to demand and partially due to deterioration. Shortages occur during the period t_0 to T and of size Q_2.

Also here $t_1 = \dfrac{Q}{\lambda}$ (1)

and defective percentage (d) is restricted to $d \le 1 - \dfrac{(a+bt-\rho p)}{\lambda}$ (2)

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

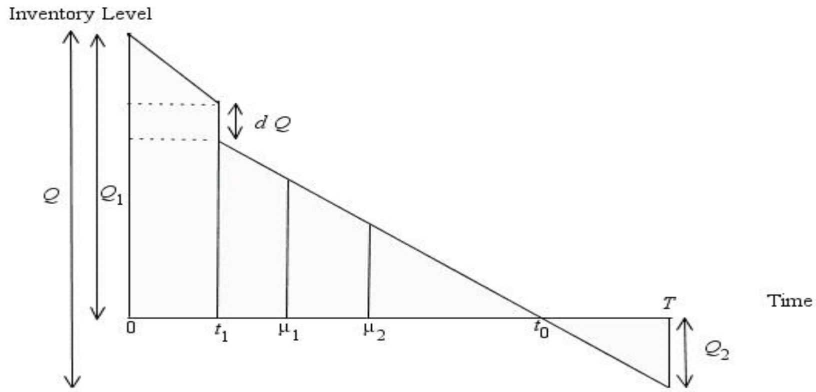

Figure 1 The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = -(a + bt - \rho p),\ 0 \le t \le \mu_1 \tag{3}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt - \rho p),\ \mu_1 \le t \le \mu_2 \tag{4}$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bt - \rho p),\ \mu_2 \le t \le t_0 \tag{5}$$

$$\frac{dI(t)}{dt} = -(a + bt - \rho p),\ t_0 \le t \le T \tag{6}$$

with initial conditions $I(0) = Q_1$, $I(\mu_1) = S_1$, $I(t_0) = 0$, and $I(T) = -Q_2$.
Solutions of these equations are given by

$$I(t) = Q_1 - (at - \rho pt + \frac{1}{2}bt^2), \tag{7}$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a\theta(\mu_1^2 - t^2) - \frac{1}{2}\rho p\theta(\mu_1^2 - t^2) + \frac{1}{2}b(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) + \rho p\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - t)\right] \tag{8}$$

$$I(t) = \begin{bmatrix} a(t_0 - t) - \rho p(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2) + \frac{1}{6}a\theta(t_0^3 - t^3) - \frac{1}{6}\rho p\theta(t_0^3 - t^3) \\ + \frac{1}{8}b\theta(t_0^4 - t^4) - \frac{1}{2}a\theta t^2(t_0 - t) + \frac{1}{2}\rho p\theta t^2(t_0 - t) - \frac{1}{4}b\theta t^2(t_0^2 - t^2) \end{bmatrix}. \tag{9}$$

$$I(t) = \left[a(t_0 - t) - \rho p(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2)\right]. \tag{10}$$

(by neglecting higher powers of θ)
After screening process, the number of defective items at time t_1 is dQ.
So effective inventory level during $t_1 \le t \le T$ is given by

$$I(t) = Q_1 - dQ - (at - \rho pt + \frac{1}{2}bt^2). \tag{11}$$

From equation (7), putting $t = \mu_1$, we have

$$Q_1 = S_1 + \left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2\right). \tag{12}$$

From equations (8) and (9), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) - \rho p(\mu_1 - \mu_2) + \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) - a\theta\mu_2(\mu_1 - \mu_2) + \rho p\theta\mu_2(\mu_1 - \mu_2) - \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - \mu_2)\right] \tag{13}$$

$$I(\mu_2) = \begin{bmatrix} a(t_0 - \mu_2) - \rho p(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(t_0^3 - \mu_2^3) \\ + \frac{1}{8}b\theta(t_0^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) \end{bmatrix}. \quad (14)$$

So from equations (13) and (14), we get

$$S_1 = \frac{1}{\left[1 + \theta(\mu_1 - \mu_2)\right]} \begin{bmatrix} a(t_0 - \mu_2) - \rho p(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(t_0^3 - \mu_2^3) + \frac{1}{8}b\theta(t_0^4 - \mu_2^4) \\ -\frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) - a(\mu_1 - \mu_2) + \rho p(\mu_1 - \mu_2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ + \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) - \rho p\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix}. \quad (15)$$

Putting value of S_1 from equation (15) into equation (8), we have

$$I(t) = \frac{\left[1 + \theta(\mu_1 - t)\right]}{\left[1 + \theta(\mu_1 - \mu_2)\right]} \begin{bmatrix} a(t_0 - \mu_2) - \rho p(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(t_0^3 - \mu_2^3) + \frac{1}{8}b\theta(t_0^4 - \mu_2^4) \\ -\frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) - a(\mu_1 - \mu_2) + \rho p(\mu_1 - \mu_2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ + \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) - \rho p\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} \quad (16)$$

$$+ \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a\theta(\mu_1^2 - t^2) - \frac{1}{2}\rho p\theta(\mu_1^2 - t^2) + \frac{1}{2}b(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) + \rho p\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix}.$$

Similarly putting value of S_1 from equation (15) in equation (12), we have

$$Q_1 = \frac{1}{\left[1 + \theta(\mu_1 - \mu_2)\right]} \begin{bmatrix} a(t_0 - \mu_2) - \rho p(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(t_0^3 - \mu_2^3) + \frac{1}{8}b\theta(t_0^4 - \mu_2^4) \\ -\frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) - a(\mu_1 - \mu_2) + \rho p(\mu_1 - \mu_2) - \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) \\ + \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}b(\mu_1^2 - \mu_2^2) - \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) + a\theta\mu_2(\mu_1 - \mu_2) - \rho p\theta\mu_2(\mu_1 - \mu_2) + \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} \quad (17)$$

$$+ \left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2\right).$$

Using (17) in (7), we have

$$I(t) = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\begin{bmatrix} a\left(t_0-\mu_2\right)-\rho p\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(t_0^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ +\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)-\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{bmatrix} + \left(a\left(\mu_1-t\right)-\rho p\left(\mu_1-t\right)+\frac{1}{2}b\left(\mu_1^2-t^2\right)\right) \tag{18}$$

Similarly putting t = T in equation (10), we have

$$Q_2 = \left[a\left(T-t_0\right)-\rho p\left(T-t_0\right)+\frac{1}{2}b\left(T^2-t_0^2\right)\right]. \tag{19}$$

Putting value of Q_1 and Q_2 from equations (17) and (19), we get value of Q.

$$Q = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\begin{bmatrix} a\left(t_0-\mu_2\right)-\rho p\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(t_0^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ +\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)-\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{bmatrix} + \left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2\right)+\left[a\left(T-t_0\right)-\rho p\left(T-t_0\right)+\frac{1}{2}b\left(T^2-t_0^2\right)\right]. \tag{20}$$

Putting value of Q_1 and Q in equation (11), we have

$$I(t) = \frac{(1-d)}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]}\begin{bmatrix} a\left(t_0-\mu_2\right)-\rho p\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(t_0^3-\mu_2^3\right)+\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right) \\ -\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right)-a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right) \\ +\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right)-\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)-\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{bmatrix} + (1-d)\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2\right)-d\left[a\left(T-t_0\right)-\rho p\left(T-t_0\right)+\frac{1}{2}b\left(T^2-t_0^2\right)\right]-\left(at-\rho pt+\frac{1}{2}bt^2\right). \tag{21}$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (22)

(ii) Screening cost (SrC) = zQ (23)

(iii) $HC = \int_0^{t_0}(x+yt)I(t)dt$

$$= \int_0^{t_1}(x+yt)I(t)dt + \int_{t_1}^{\mu_1}(x+yt)I(t)dt + \int_{\mu_1}^{\mu_2}(x+yt)I(t)dt + \int_{\mu_2}^{t_0}(x+yt)I(t)dt \tag{24}$$

(iv) $DC = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)dt + \int_{\mu_2}^{t_0}\theta tI(t)dt\right)$ (25)

(v) $SC = -c_2 \int_{t_0}^{T} I(t)dt$ (26)

(vi) $SR = \left(p\int_{0}^{T} (a + bt - \rho p)dt + p_d dQ \right).$ (27)

The total profit (π) during a cycle consisted of the following:

$$\pi = \frac{1}{T}\left[SR - OC - SrC - HC - DC - SC\right] \quad (28)$$

Substituting values from equations (22) to (27) in equation (28), we get total profit per unit. Putting $\mu_1 = v_1 t_0$ and $\mu_2 = v_2 t_0$ and value of t_1 and Q in equation (28), we get profit in terms of t_0, T and p. Differentiating equation (21) with respect to t_0, T and p and equate it to zero, we have

i.e. $\frac{\partial \pi(t_0,T,p)}{\partial t_0} = 0,\ \frac{\partial \pi(t_0,T,p)}{\partial T} = 0,\ \frac{\partial \pi(t_0,T,p)}{\partial p} = 0$ (29)

provided it satisfies the condition

$$\begin{vmatrix} \frac{\partial \pi^2(t_0,T,p)}{\partial t_0^2} & \frac{\partial \pi^2(t_0,T,p)}{\partial t_0 T} & \frac{\partial \pi^2(t_0,T,p)}{\partial t_0 p} \\ \frac{\partial \pi^2(t_0,T,p)}{\partial T t_0} & \frac{\partial \pi^2(t_0,T,p)}{\partial T^2} & \frac{\partial \pi^2(t_0,T,p)}{\partial T p} \\ \frac{\partial \pi^2(t_0,T,p)}{\partial p t_0} & \frac{\partial \pi^2(t_0,T,p)}{\partial p T} & \frac{\partial \pi^2(t_0,T,p)}{\partial p^2} \end{vmatrix} > 0 \quad (30)$$

4. NUMERICAL EXAMPLE

Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ= 10000, ρ= 5, θ=0.05, x = Rs. 5, y=0.05, v_1 = 0.30, v_2 = 0.50, c_2 = Rs. 8, in appropriate units. The optimal values of t_0* = 0.2991, T* =0.5010, p* = 50.4521, and Profit*= Rs. 12076.8484.

The second order conditions given in equation (30) are also satisfied. The graphical representation of the concavity of the profit function is also given.

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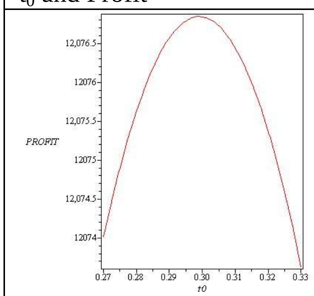
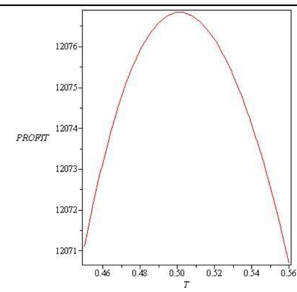
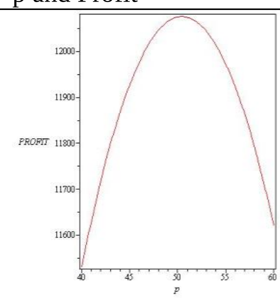
<tr><td>

Graph 1</td><td>

Graph 2</td><td>

Graph 3</td></tr>
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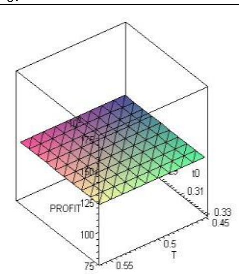
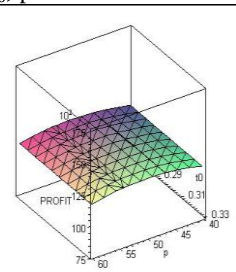
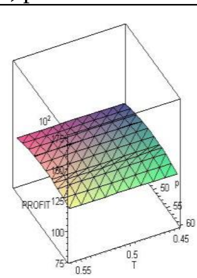
<tr><td>t_0, T and Profit</td><td>t_0, p and Profit</td><td>T, p and Profit</td></tr>
<tr><td>

Graph 4</td><td>

Graph 5</td><td>

Graph 6</td></tr>
</table>

5. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1 Sensitivity Analysis

Para-meter	%	t_0	T	p	Profit	Q
a	+20%	0.2731	0.4572	60.4168	17533.7266	136.3714
	+10%	0.2852	0.4776	55.4332	14679.8502	130.4712
	-10%	0.3152	0.5283	45.4740	9724.8588	117.7847
	-20%	0.3342	0.5606	40.5002	7624.0594	110.8894
x	+20%	0.2658	0.4778	50.4726	12056.9327	118.4513
	+10%	0.2813	0.4886	50.4629	12066.4167	121.1453
	-10%	0.3194	0.5155	50.4401	12088.3837	127.9332
	-20%	0.3429	0.5325	50.4267	12101.2334	132.2132
θ	+20%	0.2959	0.4987	50.4534	12075.2064	123.7393
	+10%	0.2975	0.4998	50.4527	12076.0236	124.0001
	-10%	0.3007	0.5023	50.4514	12077.6808	124.5942
	-20%	0.3024	0.5035	50.4507	12078.5210	124.8786
A	+20%	0.3272	0.5486	50.4906	12038.7425	136.0076
	+10%	0.3135	0.5253	50.4718	12057.3640	130.2711
	-10%	0.2839	0.4755	50.4314	12097.3287	117.9989
	-20%	0.2679	0.4484	50.4095	12118.9765	111.3139
ρ	+20%	0.2993	0.5014	42.1188	9993.6140	124.1571
	+10%	0.2992	0.5012	45.9066	10940.5295	124.2212
	-10%	0.2990	0.5009	56.0076	13465.7053	124.3735
	-20%	0.2989	0.5008	62.9521	15201.8018	124.4617
λ	+20%	0.2991	0.5011	50.4520	12076.8814	124.3099
	+10%	0.2991	0.5011	50.4520	12076.8664	124.3099
	-10%	0.2991	0.5010	50.4521	12076.8264	124.3096
	-20%	0.0.2990	0.5010	50.4522	12076.7989	124.3094
c_2	+20%	0.3093	0.4836	50.4660	12062.6763	119.9523
	+10%	0.3045	0.4916	50.4595	12069.3035	121.9440
	-10%	0.2928	0.5123	50.4435	12085.5209	127.0993
	-20%	0.2855	0.5261	50.4335	12095.6027	130.5372

From the table we observe that as parameter a increases/ decreases average total profit and optimum order quantity also increases/ decreases.

Also, we observe that with increase and decrease in the value of θ, x, c_2 and ρ, there is corresponding decrease/ increase in total profit and optimum order quantity.

From the table we observe that as parameter A increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases.

From the table we observe that as parameter λ increases/ decreases, there is almost no change in average total profit and optimum order quantity.

6. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with price and time dependent demand with different deterioration rates and shortages. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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